Diagnostic décentralisé à l'aide d'automates cellulaires

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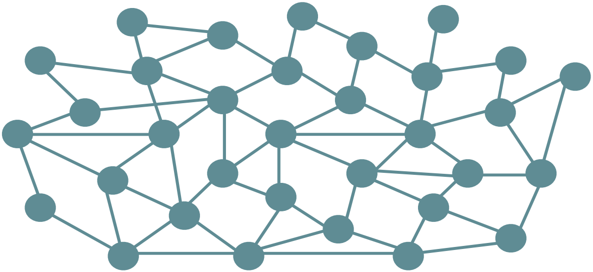
Science des Réseaux

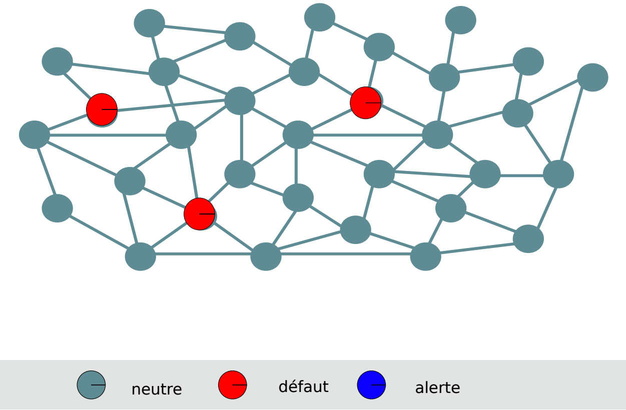
Jeudi 7 octobre 2021

UNIVERSITÉ DE LORRAINE

Diagnostic décentralisée : présentation



Diagnostic décentralisée : présentation



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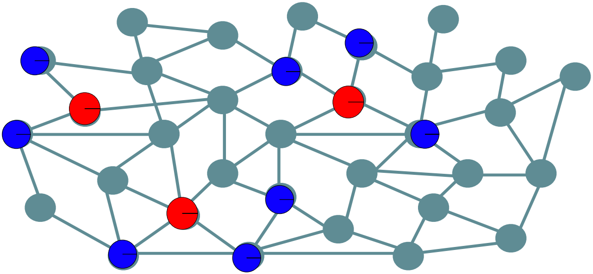

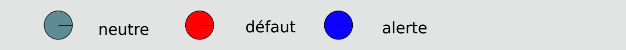

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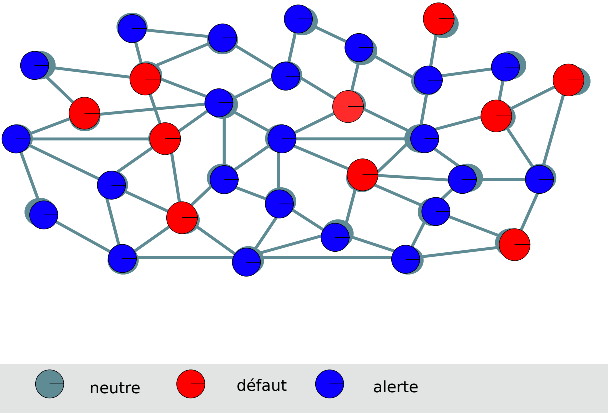
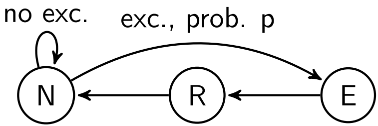


point de départ : modèle Greenberg-Hastings

pour simplifier :

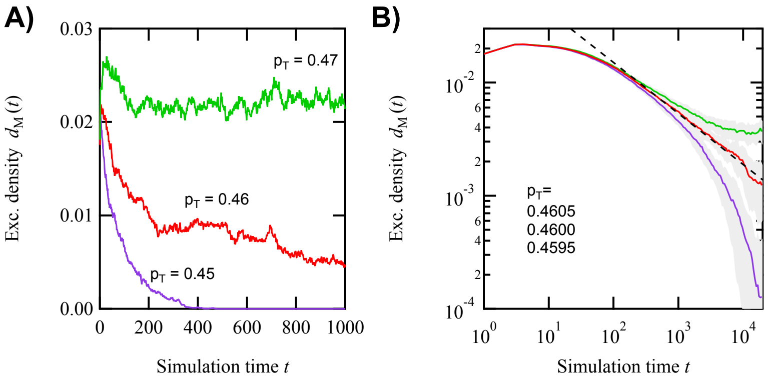
grille finie $\mathcal{L} = \{1, \ldots, X\} \times \{1, \ldots, Y\}$; simu : espace torique



états $Q = \{N, E, R\}$

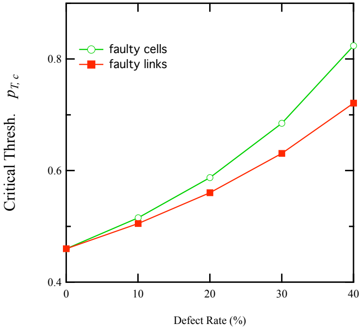
- `neutre` $\rightarrow$ `excité avec prob.` p_{T} `si au moins une cellule excitée dans son voisinage, neutre sinon`
- `excité devient réfractaire`
- `réfractaire devient neutre`

ondes qui ne s'atténuent pas ; dynamique d'épidémie (S-I-R), communication cellulaire, hola, etc.



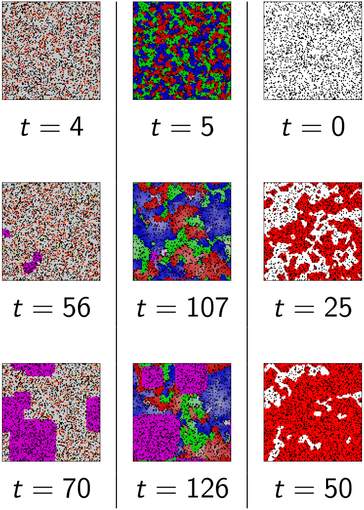
Évolution de la densité de cellules excitées $d_M(t)$ pour $M = 4$
(A) Un seul échantillon, $L = 400$. (B) Courbe en échelle log-log avec des moyennes pour 30 échantillons ; $L = 800$
pointillés indiquent percolation orientée : $d_M(t) \sim t^{-0.451}$.

sensibilité à la topologie



corrélation :
taux de défauts dans la grille et seuil critique

question : peut-on inverser le processus ?
Peut-on avoir un changement qualitatif de comportement pour un taux de cellules défaillantes donné ?



stage de Master N. Gauville

QuorumD

`Diagnostic décentralisé à l'aide d'automates cellulaires`
`Nicolas Gauville, Nazim Fatès, Irène Marcovici`
`Proceedings of JFSMA'19 , Toulouse (France), 2019, p. 96-105`

Problème du diagnostic décentralisé

Grille $\mathbb{Z}^2$, sur laquelle chaque cellule peut prendre 3 états :

- **N** : normal,
- **D** : défaillant (il s'agit d'un état fixe),
- **A** : alerte.

Objectif

Chercher une règle locale la plus simple possible telle qu'à partir d'une configuration initiale avec des cellules **N** et **D**, l'état A envahit la grille si et seulement la proportion d'états **D** dépasse un certain seuil (qu'on souhaite pouvoir choisir).

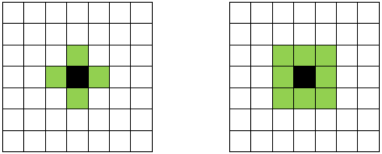
Première règle proposée

- Si toutes les voisines sont **N**, le nouvel état est **N**.
- Si toutes les voisines sont **A** ou **D**, le nouvel état est **A**.
- Sinon, le nouvel état de la cellule est :

$$\textbf{N} \text{ avec proba. } \frac{\exp(\lambda n_N)}{\exp(\lambda n_N) + \exp(\lambda(n_A + n_D))}$$

$$\textbf{A} \text{ avec proba. } \frac{\exp(\lambda(n_A + n_D))}{\exp(\lambda n_N) + \exp(\lambda(n_A + n_D))},$$

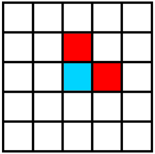
où n_N, n_A, n_D sont resp. les nb. de voisines **N**, **A**, **D**.



En ajustant la valeur de λ, on peut détecter différent seuils de défaillance.

Simplifications

Même règle, mais sur le voisinage de Toom.



On simplifie encore un peu...

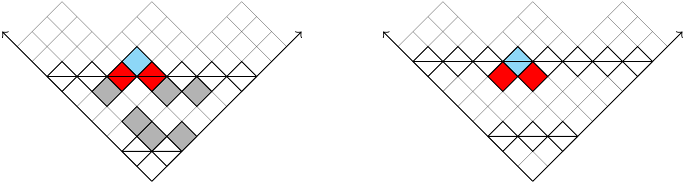
- Si les deux voisines (rouges) sont **N**, le nouvel état est **N**.
- Si les deux voisines (rouges) sont **A** ou **D**, le nouvel état est **A**.
- Sinon, on garde l'ancien état avec proba. p, et on le change avec proba. $1 - p$.

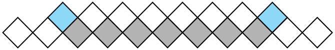
Résultats expérimentaux et analytiques

Config. init.	Cas $p < 1/2$	Cas $p > 1/2$
Nb fini de **A** et pas de **D**	$\mathbb{P}$(survie des **A**)> 0	Extinction des **A**
Nb fini de **D** et **A**	Chaîne de Markov transiente	Chaîne de Markov récurrente
Densité ρ de **A** et $1-\rho$ de **N**	Évolution vers un damier **A** / **N**	$\rho > 1/2$: extinction de **N** $\rho < 1/2$: extinction de **A**
Densité ρ de **D** et $1-\rho$ de **N**		$\rho > \rho_c(p)$: extinction de **N** $\rho < \rho_c(p)$: survie de **N**

Nombre fini de cellules en **A**lerte

- **Etat initial:** nombre fini de cellules en **A**lerte,
aucune cellule **D**éfaillante.
- **Nombre de lignes contenant des cellules A:** décroissant.



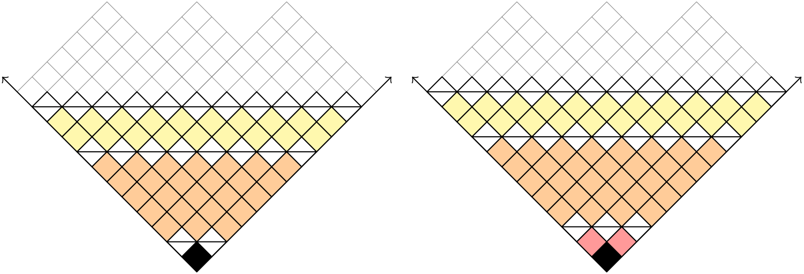
- **Longueur de la ligne frontale:** 
domination stochastique par une marche aléatoire de pas

$$(1-p)^2\delta_1 + 2p(1-p)\delta_0 + p^2\delta_{-1}$$

- **Conclusion:**
si $p > 1/2$, extinction des **A** en un temps T intégrable,
si $p < 1/2$, persistence des **A** avec probabilité strictement positive.

Une seule cellule **D**éfaillante (1)

- **Observation.** Les lignes vides montent.



- On regarde donc les $(T_i)_i$: création de lignes vides en 0.

Les $(T_{i+1} - T_i)_i$ sont iid et admettent un moment exponentiel.

- On appelle **bulle numéro** i la partie du processus qui évolue

entre les lignes créés en T_i et T_{i+1}.

Les bulles évoluent de façon indépendante.

Une seule cellule Défaillante (2)

La bulle numéro k nait à l'instant T_k, vit un temps V_k, puis disparait.

Description par une file d'attente $(R_t)_{t\geq 1}$: $R_0 = 0$ et

$$\begin{cases} \text{si } t = T_k \text{ alors} & R_{t+1} = \max(R_t - 1, V_k), \\ \text{sinon} & R_{t+1} = \max(R_t - 1, 0). \end{cases} \qquad (1)$$

Lemme

Si le temps de vie V_1 d'une bulle est **intégrable**,
alors la file d'attente est **récurrente positive**,
et le nombre de **A** *est infiniment souvent nul.*

Démonstration. Critère de Foster.

Théorème

Si $p > 1/2$, alors le temps de vie V_1 d'une bulle est **intégrable**,
et le nombre de **A** *est infiniment souvent nul.*

Temps de vie d'une bulle ($p > 1/2$)

- Le nombre N de lignes admet des moments exponentiels
- longueur de la ligne 1 $\preceq$ marche aléatoire biaisée:

disparition à l'instant H_1 intégrable.

- Tant que la ligne 1 existe, elle favorise la persistence de la ligne 2.

$$r_n^2 - r_n^1 \preceq \text{ marche aléatoire symétrique,}$$

donc contrôle de la longueur de la ligne 2 à l'instant H_1:

$$\mathbb{E}(\ell_2) \leq A\mathbb{E}(\sqrt{H_1}) + B.$$

Ensuite, elle évolue comme la ligne 1, et disparait à l'instant H_2:

$$\mathbb{E}(H_2) - \mathbb{E}(H_1) \leq A'\mathbb{E}(\sqrt{H_1}) + B'.$$

- récurrence: $\mathbb{E}(H_i) \leq A'' i^2$.
- $\mathbb{E}(V_1) = \mathbb{E}(H_N) = \sum_i \mathbb{E}(H_i)\mathbb{P}(T = i) < +\infty$ Gagné !

Network science and dynamic graphs

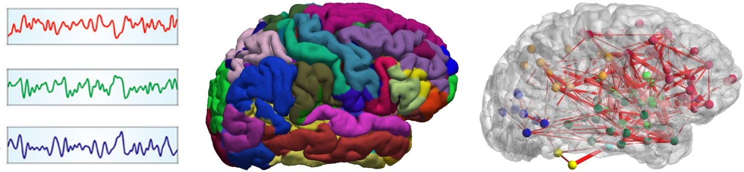
Sebastian Miron[1], Konstantin Usevich [1], Marianne Clausel[2]

[1] *Centre de Recherche en Automatique*
[2] *Institut Élie Cartan de Lorraine*

Journée FCH « Sciences des réseaux », 7 octobre 2021

Motivation

Dynamic networks are ubiquitous...



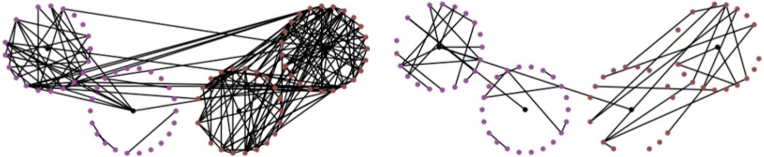
Example 1 : fMRI and brain connectivity [Achard et al. 2007] [1]

1. Achard S, Bullmore E. Efficiency and cost of economical brain functional networks. PLoS computational biology. 2007 Feb;3(2):e17.

Motivation

Dynamic networks are ubiquitous...

Contacts in primary school



Example 2 : dynamic networks and socio patterns [Gauvin et al., 2014] [2]

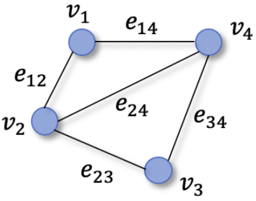
2. `http://www.sociopatterns.org/gallery/dynamical-contact-patterns-in-a-primary-school/`

Outline

Challenges

Static networks representations

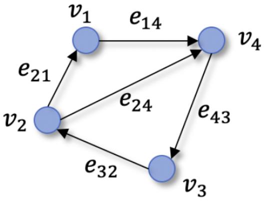
Undirected graph G(V,E)



	v_1	v_2	v_3	v_4
v_1	0	1	0	1
v_2	1	0	1	1
v_3	0	1	0	1
v_4	1	1	1	0

binary symmetric

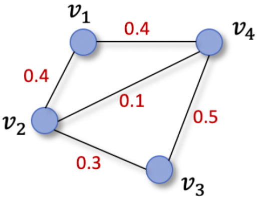
Directed graph G(V,E)



	v_1	v_2	v_3	v_4
v_1	0	0	0	1
v_2	1	0	0	1
v_3	0	1	0	0
v_4	0	0	1	0

binary non-symmetric

Weighted graph G(V,E)



	v_1	v_2	v_3	v_4
v_1	0	0.4	0	0.4
v_2	0.4	0	0.3	0.1
v_3	0	0.3	0	0.5
v_4	0.4	0.1	0.5	0

real-valued

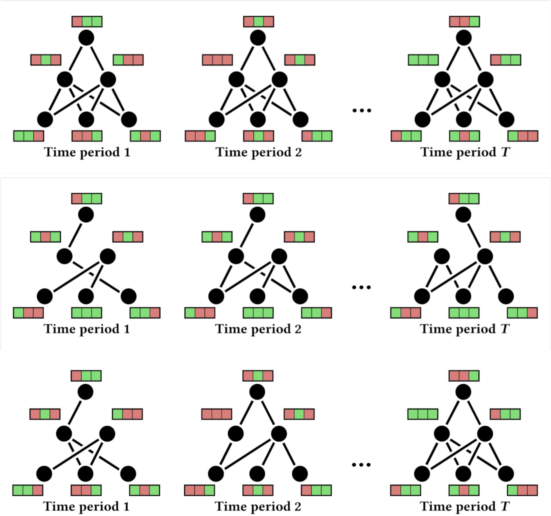
Modeling dynamic networks ?

Graph *vs.* multivariate time series

How can we take into account evolution with time ?

- the value of each node
- the dependence between the nodes
- the topology of the network can evolve with time

Modeling dynamic networks ?



Static graph with dynamic signal

Dynamic graph with static signal

Dynamic graph with dynamic signal

Scientific questions

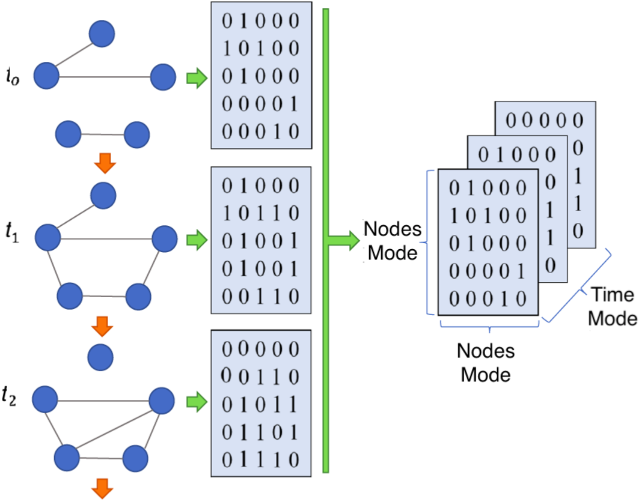
Dynamic networks for practitioners

Practical issues

- Can we group the nodes into relevant categories ?
- Structural changes
- Novelty and anomaly detection

Tensor approach for dynamic graphs

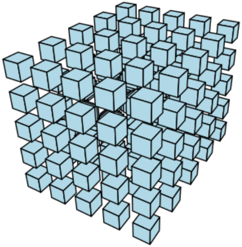
Tensors : a natural model for dynamic network



Tensors : a natural model for dynamic network

Tensors and dynamic networks analysis

- Classical static graph analysis is based on the study of spectral properties of the adjacency matrix/Laplacian
- Taking into account time needs more complex objects : tensors



- Many challenges with tensors [3] [4]
  - storage
  - no straightforward extension of classical spectral analysis !

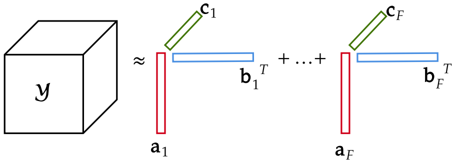
3. T. G. Kolda, B. W. Bader. Tensor Decompositions and Applications. SIAM Review, 2009.
4. P. Comon, Tenseurs en Sciences de Données (Encyclopédie des Techniques de l'Ingénieurs), 2021.

Tensors : a natural model for dynamic network

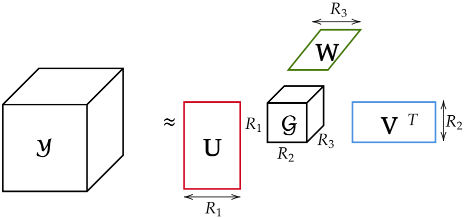
Main challenges

- compression of information
- dynamic clustering of nodes
- tensor decompositions :
  - possibility to take into account specific constraints (positivity, low rank) related to the application
  - existence of theoretical guarantees
  - efficient optimisation algorithms for decomposition

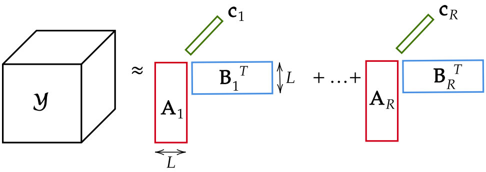
Tensor decompositions



Canonical Polyadic (CP) Decomposition

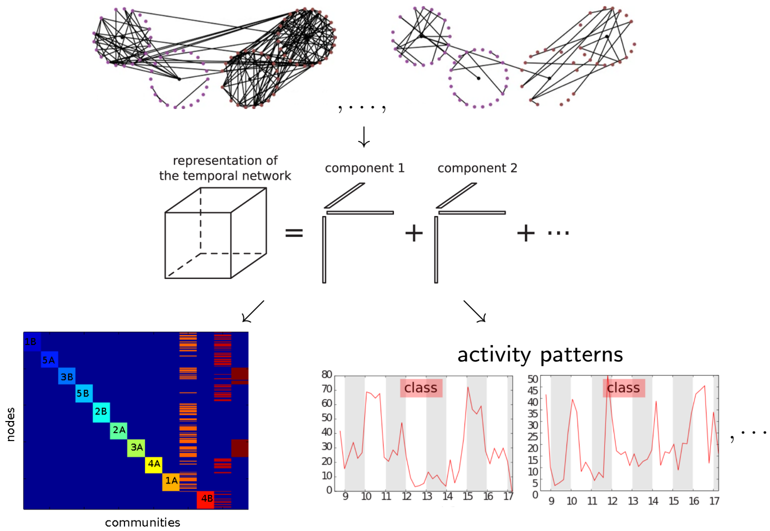


Tucker Decomposition



LL1 (block-term) decomposition

Example : contacts in a school



- [Gauvin et al., 2014] : nonnegative CP decomposition
- PhD thesis of M. Diop (CRAN, 2018) : binary tensor decomposition

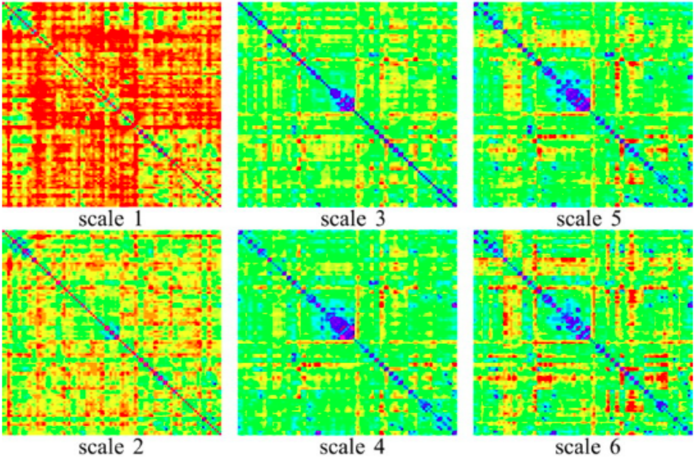
Multivariate time series approach for dynamic graphs

Multivariate time series

More about dependence modeling

- First approach to model dynamic dependence between nodes : adjacency/weights matrix
- Alternative approach : nodes are time series
- Several notion of dependence are involved
- Time-dependence and dependence between components can be combined
- Several way to model dependence between time series : empirical correlation, long-run correlation, wavelet correlation, signature of order 2...

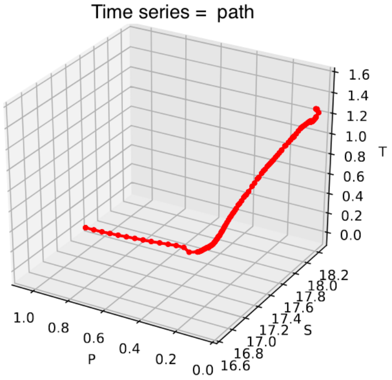
Multivariate time series

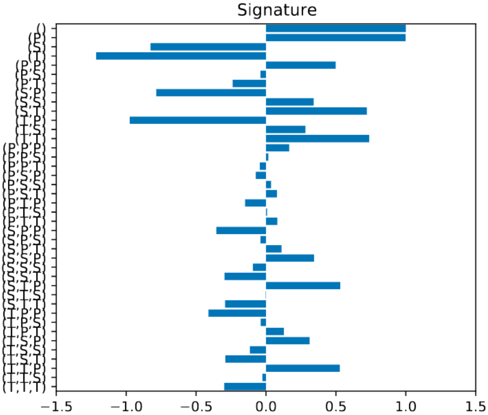


An example of dependence measure : wavelet correlation [5]

5. S. Achard et al., A Resilient, Low-Frequency, Small-World Human Brain Functional Network with Highly Connected Association Cortical Hubs

Multivariate time series





An example of dependence measure : signature coefficients [6]

6. N. Sugiura, Machine Learning Technique Using the Signature Method for Automated Quality Control of Argo Profiles

Conclusion

Perspectives

- Dynamic networks : interdisciplinary field with challenging questions !
- Beyond graphs
  - Model dependence with more than two objects ?
  - Analyze information transfer and causality in oriented graphs

Laplacian Spectra of Graphs and Cyber-Insurance Protection

Nabil Kazi-Tani
IECL, Université de Lorraine
En collaboration avec Thierry Cohignac (CCR, Paris)

Journée Science des Réseaux - 7 octobre 2021

Introduction

- A given graph has to be protected against communication interruption, via prevention measures, or via insurance.
- Graph $=$ Company's computer network, susceptible to exterior attacks.

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- Or... Interconnected data, in the form of a graph $\rightarrow$ susceptible to data breaches, which can be insured.

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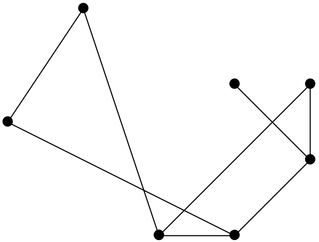
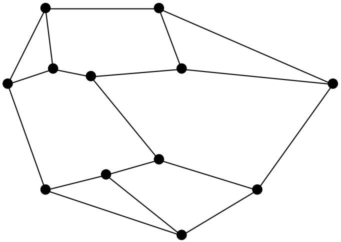
- A given graph has to be protected against communication interruption, via prevention measures, or via insurance.
- Graph = Company's computer network, susceptible to exterior attacks.
- Or... Interconnected data, in the form of a graph $\rightarrow$ susceptible to data breaches, which can be insured.
- Question: impact of the topology of the network on the chosen protection?

Introduction

- **Nodes** can represent: computers, servers, softwares, routers, data sets, etc.
- **Edges** can be: internet connections, distant access to servers, links between softwares etc.



(a) A graph on $n = 7$ vertices, with 8 edges.



(b) A 3-regular graph on $n = 12$ vertices.

Inhomogeneous Suseptible-Infected-Susceptible (SIS) model on Edges.

- State at time t of edge i is Bernoulli: $X_i(t) = 0$ when edge i is **healthy** and $X_i(t) = 1$ when edge i is infected.

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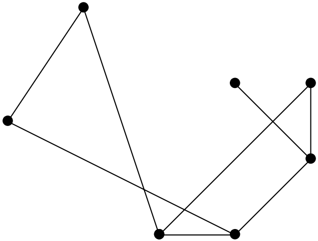
- When infected, edge i can infect each edge j sharing a common vertex with rate $\beta_{ij} > 0$.
- Each edge i can be cured, i.e. its state X_i jumps from 1 to 0 at rate $\delta_i > 0$.
- The Poisson infection and curing processes are assumed independent.

Epidemic model

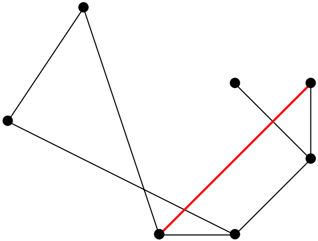
Inhomogeneous Suseptible-Infected-Susceptible (SIS) model.



No edge is infected.

Epidemic model

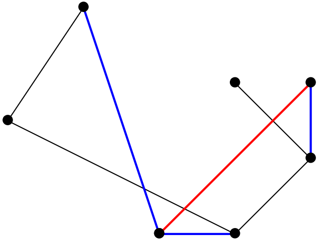
Inhomogeneous Suseptible-Infected-Susceptible (SIS) model.



Red edge i is infected, and can be cured at rate $\delta_i > 0$.

Epidemic model

Inhomogeneous Suseptible-Infected-Susceptible (SIS) model.



Edge i infects each blue edge j at rate $\beta_{ij} > 0$.

Epidemic model

- We use 2 simplifying approximations:

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- N-Intertwined **Mean Field Approximation** (Kooij, Omic, Van Mieghem 2008): upper bounds the infection probabilities.

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- N-Intertwined **Mean Field Approximation** (Kooij, Omic, Van Mieghem 2008): upper bounds the infection probabilities.
- **Long time steady state**: which is accurate when infection probabilities are high, corresponding to **cascading infections**.

Epidemic model - Approximations

Let $\widetilde{a} =$ edge adjacency matrix and $m =$ number of edges.

$$\frac{X_i(t+\Delta t) - X_i(t)}{\Delta t} = (1 - X_i(t)) \sum_{j=1}^{m} \tilde{a}_{ij} \beta_{ij} X_j(t) - \delta_i X_i(t).$$

Epidemic model - Approximations

Let $\widetilde{a} =$ edge adjacency matrix and $m =$ number of edges.

$$\frac{X_i(t+\Delta t) - X_i(t)}{\Delta t} = (1 - X_i(t)) \sum_{j=1}^{m} \tilde{a}_{ij}\beta_{ij}X_j(t) - \delta_i X_i(t).$$

leads to

$$\frac{dv_i(t)}{dt} = \sum_{j=1}^{m} \tilde{a}_{ij}\beta_{ij}v_j(t) - \sum_{j=1}^{m} \tilde{a}_{ij}\beta_{ij}\mathbb{E}[X_i(t)X_j(t)] - \delta_i v_i(t),$$

where $v_i(t) = \mathbb{P}(X_i(t) = 1)$.

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N-Intertwined Mean Field Approximation:

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i.e

$$\frac{dv_i(t)}{dt} = \left(\sum_{j=1}^{m} \tilde{a}_{ij}\beta_{ij} v_j(t)\right)(1 - v_i(t)) - \delta_i v_i(t), \quad i = 1, \ldots, m.$$

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N-Intertwined Mean Field Approximation and **Long time steady state**:

$$\frac{dv_i(t)}{dt} = 0 = \left(\sum_{j=1}^{m} \tilde{a}_{ij}\beta_{ij}v_j(t)\right)(1 - v_i(t)) - \delta_i v_i(t), \quad i = 1, \ldots, m.$$

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i.e.

$$v_i = \frac{\beta \sum_{j=1}^{m} \tilde{a}_{ij} v_j}{\delta_i + \beta \sum_{j=1}^{m} \tilde{a}_{ij} v_j}, \quad i = 1, \dots m,$$

where $\beta_{ij} = \beta$.

Optimization Problem

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- *Stand Alone Cyber Contracts*: clauses of repairing actions, through the intervention of a specified cyber security company.
- Prevention or *cyber hygiene* measures: back-ups, users training, network mapping and inventory etc.

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- *Stand Alone Cyber Contracts*: clauses of repairing actions, through the intervention of a specified cyber security company.
- Prevention or *cyber hygiene* measures: back-ups, users training, network mapping and inventory etc.
- How should we measure the "connectedness" of a given graph G?

Optimization Criteria

- Given a graph G with adjacency matrix A and degree matrix D, the Laplacian matrix of G is given by $\mathrm{L} := \mathrm{D} - \mathrm{A}$.

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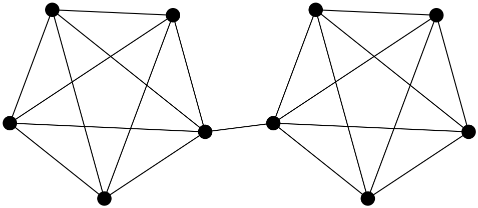
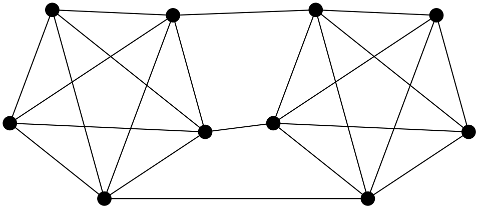
- **Algebraic connectivity** $\lambda_2(G)$= lowest strictly positive eigenvalue of the Laplacian matrix.
- Known to describe the "connectedness" of a graph (Fiedler 1973).

Optimization Criteria



(a) A graph G, such that $\lambda_2(G) = 0.298$.



(b) A graph $\widetilde{G}$, such that $\lambda_2(\widetilde{G}) = 1$.

Figure: The lower graph $\widetilde{G}$ is "more connected" than the upper graph G.

Optimization Criteria

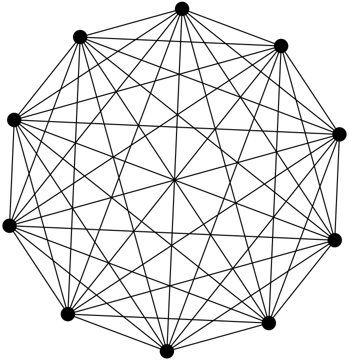


Figure: Complete graph K_{10} with $\lambda_2(K_{10}) = 10$.

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4. Maximize the algebraic connectivity of L:

$$\begin{aligned} \underset{\delta \in \mathcal{A}}{\text{maximize}} \quad & \lambda_2(L) \\ \text{subject to} \quad & c(\delta) \leq B, \end{aligned}$$

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5. (Edge i interrupted $\to$ loss Z_i with $\mathbb{E}[Z_i] = c_i$.)

A Solution

Theorem

An optimal solution is given by

$$\delta_i^\star = \left(\beta \sum_{j=1}^{m} \tilde{a}_{ij} v_j^\star \right) \frac{1 - v_i^\star}{v_i^\star},$$

where $(1 - v_1^\star, \ldots, 1 - v_m^\star)$ is the w component of a solution to the following convex problem in $\gamma \in \mathbb{R}$, $\mu \in \mathbb{R}$ and $w \in \mathbb{R}^m$:

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$$\begin{aligned} \text{sup} \quad & \gamma \\ \text{subject to} \quad & L(w) - \gamma I + \mu 1 1^t \succcurlyeq 0 \\ & 0 \leq w \leq 1 \\ & \sum_{j=1}^{m} c_j w_j \leq B, \end{aligned}$$

where $1^t := (1, \ldots, 1) \in \mathbb{R}^m$, $L(w)$ denotes the Laplacian matrix of the weighted graph (V, E_0, w), $M \succcurlyeq 0$ means that M is positive semidefinite and $(\tilde{a}_{ij})_{a \leq i,j \leq m}$ are the entries of the edge-adjacency matrix.

Examples

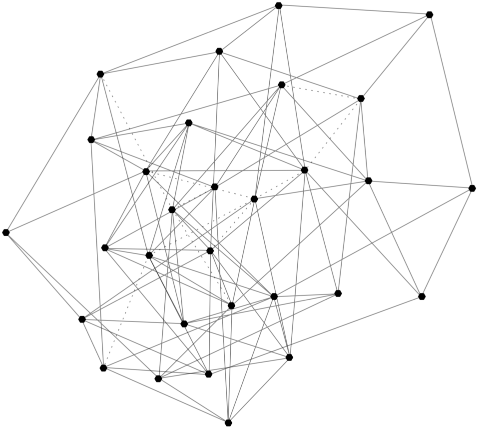

Figure: Erdös-Rényi Random Graph with 102 edges. The dotted edges have optimal δ equal to 0.

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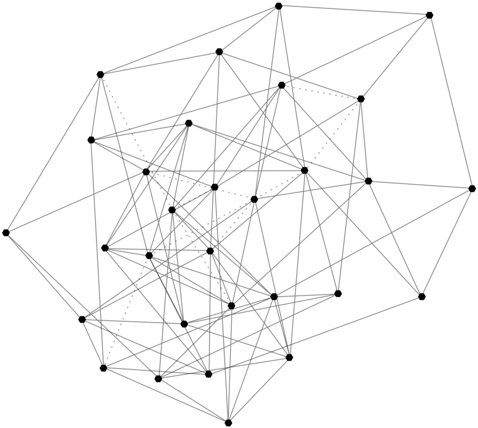

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Solved with the **CVX Matlab Package (Boyd, Grant)**.

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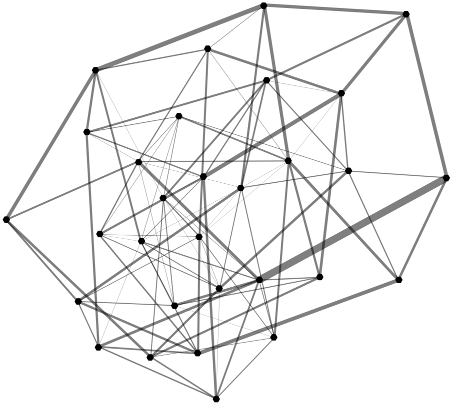

Figure: Each edge width is proportional to the optimal δ.

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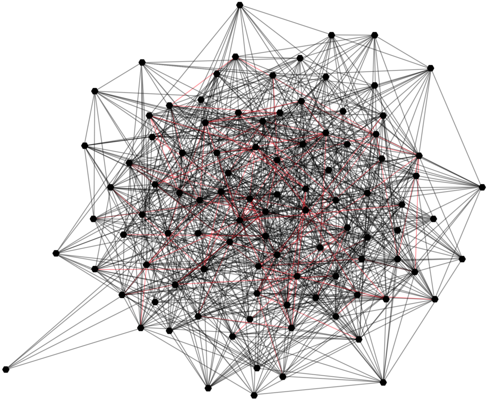
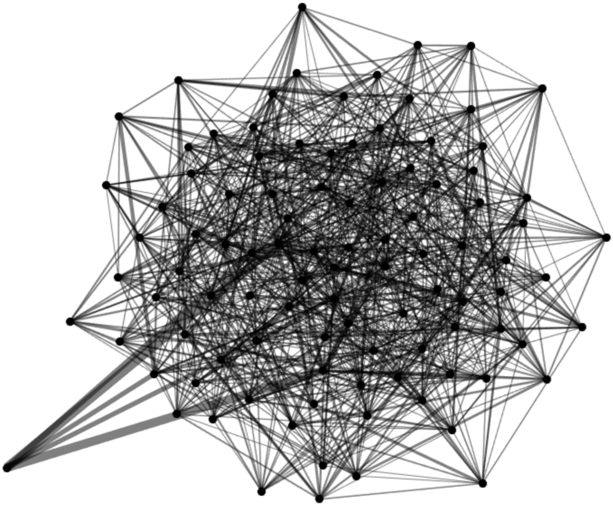


Figure: Erdös-Rényi Random Graph with 1004 edges. Red edges have null $\delta^\star$ values.

Examples



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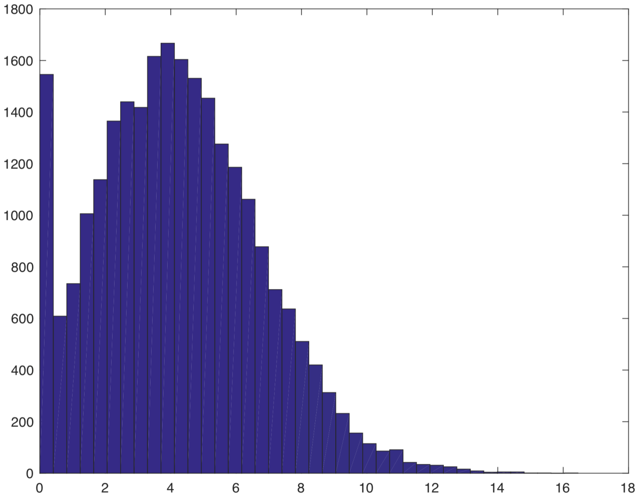

Figure: $\delta^{\star}$ values, for a graph with 24978 edges.

Perspectives

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- Inhomogeneous fast mixing Markov chains (Boyd, Diaconis, Sun, Xiao).
- Simulation and (optimal) importance sampling for Markov chains.
- Optimal stochastic control and games on networks.

Merci de votre attention